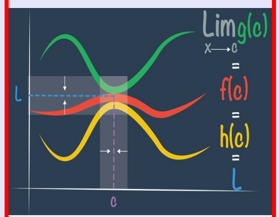
Calculus I

Lecture 11



Feb 19-8:47 AM

Open Notes

Class QZ 11

$\sqrt{5} \approx \sqrt{4}$

a

Use linear approximation to estimate $\sqrt{5}$.

Round to 3-dec. Places.

$\boxed{f(x)=\sqrt{x}}$ $\quad f'(x)=\frac{1}{2\sqrt{x}}$ $\quad f(x)\approx f(a)+f'(a)(x-a)$

$\boxed{a=4}$

$f(4)=2$ $\quad f'(4)=\frac{1}{4}$

$\sqrt{x}\approx 2+\frac{1}{4}(x-4)$

$\sqrt{5}\approx 2+\frac{1}{4}(5-4)$

$\approx \boxed{2.25}$

Quadratic Approximation

$f(x)\approx f(a)+f'(a)(x-a)+\frac{f''(a)}{2}(x-a)^2$

$f(x)=\sqrt{x}=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}$

$f''(x)=\frac{1}{2}\cdot\frac{-1}{2}x^{-3/2}=\frac{-1}{4}x^{-3/2}$

$=\frac{-1}{4x^{3/2}}=\frac{-1}{4x\sqrt{x}}$

$f''(4)=\frac{-1}{4\cdot 4\sqrt{4}}=\frac{-1}{32}$

$\sqrt{x}\approx 2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2$

$\sqrt{5}\approx 2+\frac{1}{4}-\frac{1}{64}$

$=2.234375$

From Calc

$\sqrt{5}\approx 2.236067977$

Jan 21-11:53 AM

Use quadratic approximation to estimate $(4.1)^3$.

By Calculator

$4.1^3 \approx 68.921$

$4.1^3 \approx 4^3 = 64$

$f(x) = x^3$

$a = 4$

$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

$x^3 \approx f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2$

$x^3 \approx 64 + 48(x-4) + 12(x-4)^2$

$f(4) = 4^3 = 64$

$f'(x) = 3x^2 \qquad f'(4) = 3(4)^2 = 48$

$f''(x) = 6x \qquad f''(4) = 6(4) = 24$

$4.1^3 \approx 64 + 48(4.1-4) + 12(4.1-4)^2$

$= 64 + 48(.1) + 12(.1)^2$

$= 64 + 4.8 + .12$

$= \boxed{68.92}$

Jan 22-8:18 AM

$f(x) = \frac{x}{x^2-1}$

1) Domain

$x \neq \pm 1$

2) Y-Int $(0,0)$

3) X-Int $(0,0)$

4) Continuity

everywhere except ± 1

5) $f(-x) = \frac{-x}{(-x)^2-1} = \frac{-x}{x^2-1} = -\frac{x}{x^2-1} = -f(x)$

$f(-x) = -f(x) \rightarrow f(x)$ is an odd function

Symmetric w/t origin.

6) $f'(x) = \frac{-(x^2+1)}{(x^2-1)^2}$

$f'(x) = 0 \qquad x^2+1 = 0$

No Soln.

$f'(x)$ is und. $\quad x^2-1=0$

$x^2 = 1 \quad x = \pm\sqrt{1}$

$x = \pm 1$

7) $f''(x) = \frac{2x(x^2+3)}{(x^2-1)^3}$

$f''(x) = 0 \qquad 2x(x^2+3) = 0$

$\hookrightarrow x = 0$

$f''(x)$ is und. at

$x^2-1=0$

$\vdots$

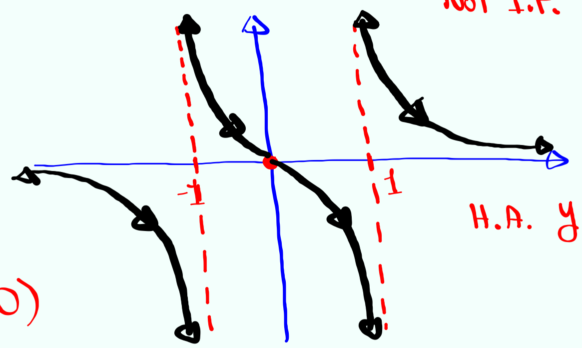
$x = \pm 1$

Jan 20-9:40 AM

$$f(x)=\frac{x}{x^2-1}\ ,\quad f'(x)=\frac{-(x^2+1)}{(x^2-1)^2}\quad f''(x)=\frac{2x(x^2+3)}{(x^2-1)^3}$$

x	$-\infty$		-1		0		1		∞
$f'(x)$		$-$	o	$-$		$-$	o	$-$	
$f''(x)$		$-$	o	$+$	●	$-$	o	$+$	
$f(x)$		↘		↘		↘		↘	

Not I.P.



H.A. $y=0$

Range
$(-\infty,\infty)$

Jan 21-9:39 AM

$$f(x)=\frac{x}{x^2-1}$$

$$f'(x)=\frac{1(x^2-1)-x\cdot 2x}{(x^2-1)^2}=\frac{x^2-1-2x^2}{(x^2-1)^2}=\frac{-x^2-1}{(x^2-1)^2}$$

$$=-\frac{x^2+1}{(x^2-1)^2}$$

$$f''(x)=-\left[\frac{2x(x^2-1)^2-(x^2+1)\cdot 2(x^2-1)\cdot 2x}{((x^2-1)^2)^2}\right]$$

$$=-\frac{2x\cancel{(x^2-1)}\left[x^2-1-2(x^2+1)\right]}{(x^2-1)^{\cancel{4}\,3}}$$

$$=-\frac{2x(x^2-1-2x^2-2)}{(x^2-1)^3}=-\frac{2x\overbrace{(-x^2-3)}^{-(x^2+3)}}{(x^2-1)^3}=\frac{2x(x^2+3)}{(x^2-1)^3}$$

Jan 22-8:30 AM

$$f(x) = \frac{x}{x^3-1}$$

$$f'(x) = \frac{1(x^3-1) - x \cdot 3x^2}{(x^3-1)^2} = \frac{x^3-1-3x^3}{(x^3-1)^2} = \frac{-2x^3-1}{(x^3-1)^2}$$

$$= \frac{-(2x^3+1)}{(x^3-1)^2}$$

$$f''(x) = -\left[\frac{6x^2(x^3-1)^2 - (2x^3+1) \cdot 2(x^3-1) \cdot 3x^2}{(x^3-1)^4}\right]$$

$$= -\frac{6x^2 \cancel{(x^3-1)}\left[x^3-1-(2x^3+1)\right]}{(x^3-1)^{\cancel{4}\,3}}$$

$$= -\frac{6x^2(x^3-1-2x^3-1)}{(x^3-1)^3} = -\frac{6x^2(-x^3-2)}{(x^3-1)^3}$$

$$= \frac{6x^2(x^3+2)}{(x^3-1)^3}$$

Jan 22-8:39 AM

C.N. $\rightarrow$ $f'(x) = 0$ or undefined

$$f'(x) = \frac{-(2x^3+1)}{(x^3-1)^2}$$

$f'(x) = 0$

$2x^3+1=0$

$2x^3 = -1$

$x^3 = \frac{-1}{2}$

$x = \sqrt[3]{\frac{-1}{2}} = -\sqrt[3]{\frac{1}{2}}$

$f'(x)$ undefined

$x^3-1=0$

$x=1$

P.I.P. $\rightarrow$ $f''(x)=0$ or undefined

$$f''(x) = \frac{6x^2(x^3+2)}{(x^3-1)^3}$$

$f''(x)=0$

$6x^2(x^3+2)=0$

$x=0 \quad x=\sqrt[3]{-2} = -\sqrt[3]{2}$

$f''(x)$ undefined

$x^3-1=0$

$x=1$

Sign chart

x	$-\infty$	$-\sqrt[3]{2}$		$-\sqrt[3]{\frac{1}{2}}$		0		1	∞
$f'(x)$	+		+	●	−		−	o	−
$f''(x)$	+	●	−		−	●	−	o	+
$f(x)$	↗		↗		↘		↘	o	↘

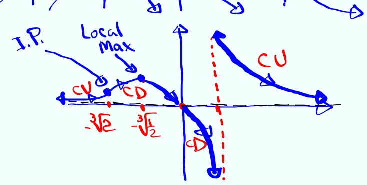
$f(x) = \frac{x}{x^3-1}$

x-Int $(0,0)$

y-Int $(0,0)$

V.A. $x=1$

H.A. $y=0$



Jan 22-8:46 AM

$$f(x) = \frac{x^3}{x-2}$$

1) Domain $x \neq 2$
$(-\infty, 2) \cup (2, \infty)$

2) VA $x - 2 = 0$
$x = 2$

3) All intercepts
y-Int $(0, 0)$, x-Int $(0, 0)$ odd times

4) $f'(x) = \dfrac{2(x-3)x^2}{(x-2)^2}$ $f''(x) = \dfrac{2x(x^2-6x+12)}{(x-2)^3}$

C.N. $3, 0, 2$ P.I.P. $0, 2$

x	$-\infty$	0		2		3	∞
f'	$-$	•	$-$	○	$-$	•	$+$
f''	$+$	•	$-$	○	$+$		$+$
f				○			

$x^2 - 6x + 12 =$

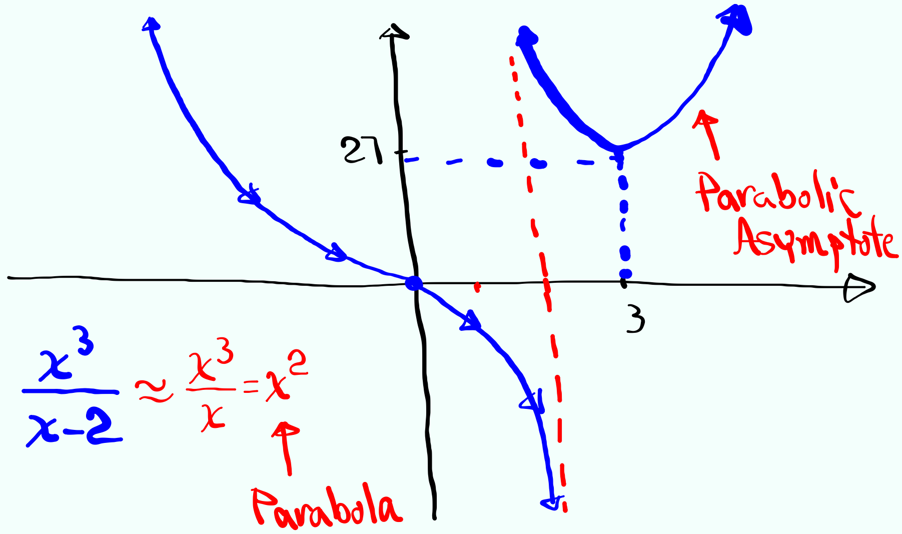
$\underbrace{x^2 - 6x + 9} + 3 =$

$(x-3)^2 + 3 > 0$

No Real Soln.

$f(3) = \dfrac{3^3}{3-2} = \dfrac{27}{1} = 27$

Jan 22-9:07 AM



$$f(x) = \frac{x^3}{x-2} \approx \frac{x^3}{x} = x^2$$

Parabola

Jan 22-9:22 AM

$f(x)=\frac{x^2}{x-2}$ ⟶ $f(4)=\frac{4^2}{4-2}=\frac{16}{2}=\boxed{8}$

1) Domain
$(-\infty,2)\cup(2,\infty)$

2) V.A. $x=2$

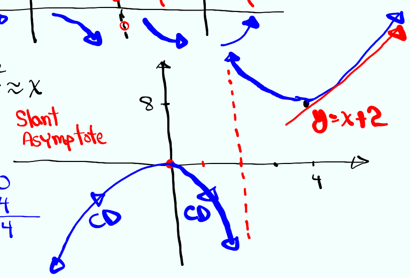
3) All intercepts   y-Int $(0,0)$ x-Int $(0,0)$ even times

4) $f'(x)=\frac{x(x-4)}{(x-2)^2}$ $f''(x)=\frac{8}{(x-2)^3}$

C.N. $0,4,2$ P.I.P. 2

x	$-\infty$ to 0	0 to 2	2 to 4	4 to ∞
f'	+	−	−	+
f''	−	−	+	+
f				

$f(x)=\frac{x^2}{x-2}\approx\frac{x^2}{x}\approx x$ Slant Asymptote



$$\begin{array}{r|rrr} 2 & 1 & 0 & 0 \\ & & 2 & 4 \\ \hline & 1 & 2 & 4 \end{array}$$

$f(x)=\boxed{x+2}+\frac{4}{x-2}$

slant Asymptote

Jan 22-9:29 AM

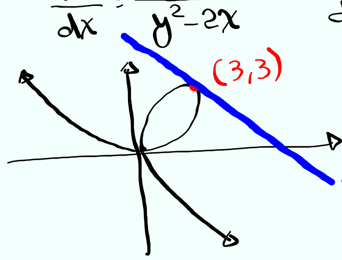
Given $x^3+y^3=6xy$

1) verify $(3,3)$ is on the graph. ✓
$3^3+3^3=6(3)(3)$ $27+27=54$

2) Find $\frac{dy}{dx}$ $3x^2+3y^2\frac{dy}{dx}=6\left(1\cdot y+x\cdot\frac{dy}{dx}\right)$
$x^2+y^2\frac{dy}{dx}=2y+2x\frac{dy}{dx}$

3) Evaluate $\left.\frac{dy}{dx}\right|_{(3,3)}$ $(y^2-2x)\frac{dy}{dx}=2y-x^2$

$\frac{dy}{dx}=\frac{2y-x^2}{y^2-2x}$ $\left.\frac{dy}{dx}\right|_{(3,3)}=\frac{2\cdot3-3^2}{3^2-2\cdot3}=\boxed{-1}$

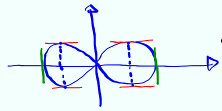


$m=\left.\frac{dy}{dx}\right|_{(3,3)}=-1$

$y-3=-1(x-3)$ $\boxed{y=-x+6}$

Jan 22-10:15 AM

Find all points on the graph of $2(x^2+y^2)^2 = 25(x^2-y^2)$ where we have horizontal and vertical tan. lines.



$$2\cdot 2(x^2+y^2)^1\cdot(2x+2yy') = 25(2x-2yy')$$

$$4(x^2+y^2)\cdot 2x + 4(x^2+y^2)\cdot 2yy' = 50x - 50yy'$$

$$(8y(x^2+y^2)+50y)y' = 50x - 8x(x^2+y^2)$$

$$y' = \frac{50x - 8x(x^2+y^2)}{50y + 8y(x^2+y^2)} = \frac{2x(25-4(x^2+y^2))}{2y(25+4(x^2+y^2))}$$

H.A. $y' = 0$

$$2x(25-4(x^2+y^2)) = 0$$

$x = 0$ $\qquad 25 - 4(x^2+y^2) = 0$

$$\begin{cases} x^2+y^2 = \frac{25}{4} \\ 2(x^2+y^2)^2 = 25(x^2-y^2) \end{cases}$$

$$2\left(\frac{25}{4}\right)^2 = 25(x^2-y^2)$$

$$\frac{625}{8} = 25(x^2-y^2)$$

$$\begin{cases} x^2+\cancel{y^2} = \frac{25}{4} \\ x^2-\cancel{y^2} = \frac{25}{8} \end{cases}$$

$$2x^2 = \frac{25}{4} + \frac{25}{8}$$

$$2x^2 = \frac{50+25}{8}$$

$$2x^2 = \frac{75}{8}$$

$$x^2 = \frac{75}{16}$$

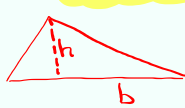
$$x = \pm\frac{5\sqrt{3}}{4}, \quad y = ?$$

Jan 22-10:27 AM

The base of a triangle is increasing at the rate of 6 in./s.

The height is decreasing at the rate of 2 in./s.

How fast the area is changing when base is 4 inches, height is 10 inches



$A = \frac{bh}{2}$ $\qquad \frac{db}{dt} = 6$ in./s. $\qquad \frac{dh}{dt} = -2$ in/s.

$$\frac{dA}{dt} = \frac{d}{dt}\left[\frac{bh}{2}\right]$$

$$= \frac{1}{2}\frac{d}{dt}[bh]$$

$$= \frac{1}{2}\left[\frac{db}{dt}\cdot h + b\cdot\frac{dh}{dt}\right]$$

$$= \frac{1}{2}\left[6\cdot 10 + 4\cdot(-2)\right] = \frac{1}{2}[60-8]$$

$$= \frac{52}{2} = \boxed{26 \text{ in}^2/\text{s}}$$

Jan 22-10:45 AM

Air is filling a spherical balloon at the rate of $3\pi \text{ cm}^3/\text{s}$. $\frac{dV}{dt} = 3\pi \text{ cm}^3/\text{s}$

How fast is its radius changing when the diameter of the balloon is 20 cm?

$V = \frac{4\pi r^3}{3}$ $\frac{dr}{dt} = ?$ $r = 10$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{4\pi r^3}{3}\right]$$

$$= \frac{4\pi}{\cancel{3}} \cdot \cancel{3}r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$3\pi = 4\pi(10)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{3}{400} \text{ cm/s}$$

Jan 22-10:53 AM

At noon, train A headed North @ 80 mph.

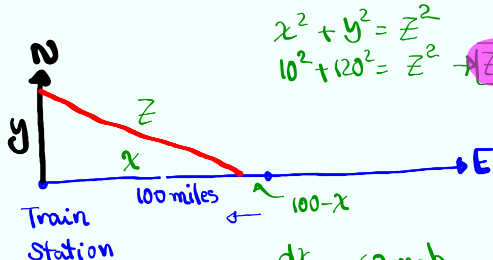
train B is 100 miles east of the train station traveling towards station at 60 mph.

what is the rate of change of the distance between them at 1:30 PM?

$x^2 + y^2 = z^2$

$10^2 + 120^2 = z^2 \rightarrow z =$

$\frac{dy}{dt} = 80$ mph



$\frac{dx}{dt} = -60$ mph

Noon → 1:30 → $y = 120$

→ $x = 10$

$x^2 + y^2 = z^2$

$$2x\boxed{\frac{dx}{dt}} + 2y\boxed{\frac{dy}{dt}} = 2z\frac{dz}{dt}$$

Jan 22-11:04 AM

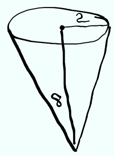
A large tank is in the shape of inverted Cone.

It is being filled with water at the rate of $\boxed{4\ m^3/hr.}$

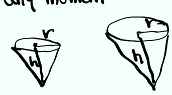
Formula for the Volume of a Cone

$$V=\frac{1}{3}\pi r^2 h$$

The original tank has a height of 8 m and radius of 2m.



At any moment



$$\frac{r}{h}=\frac{2}{8}$$

$$\frac{r}{h}=\frac{1}{4}$$

$$h=4r$$

$$V=\frac{1}{3}\pi r^2 h$$

$$V=\frac{1}{3}\pi r^2\cdot 4r \qquad V=\frac{4\pi}{3}r^3$$

How fast the radius changing when $r=6$ in?

$$\frac{dV}{dt}=\frac{4\pi}{\cancel{3}}\cdot \cancel{3}r^2\frac{dr}{dt}$$

$$4=4\pi(6)^2\frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt}=\frac{1}{36\pi}\ m/hr.}$$

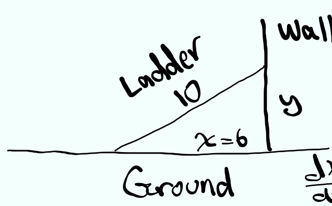
Jan 22-11:16 AM

A 10-ft ladder is leaning against a wall.

If the base slides away from the wall at 1 ft/sec., how fast is the top of the ladder is sliding down when the base is 6ft from the wall.

$$6^2+y^2=10^2$$

$$y=8$$



$\frac{dy}{dt}=-\,-$

$\frac{dx}{dt}=1$

$$x^2+y^2=10^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$6\cdot 1+8\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=\frac{-6}{8}$$

$$=\frac{-3}{4}\ \text{ft/sec.}$$

Jan 22-11:28 AM

A police helicopter is flying at the speed of 200 km/hr at a constant altitude of 1 km above a straight road.

The pilot notices a car 2km away from the helicoptor, and the distance decreasing at 250 KPh.

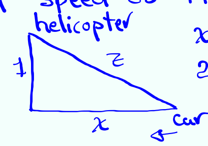
$\frac{dz}{dt} = -250$ kph

$x^2 + 1^2 = 2^2$

$x^2 = 3$

$x = \sqrt{3}$

find speed of the car.



$x^2 + 1^2 = z^2$

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

$\frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt}$

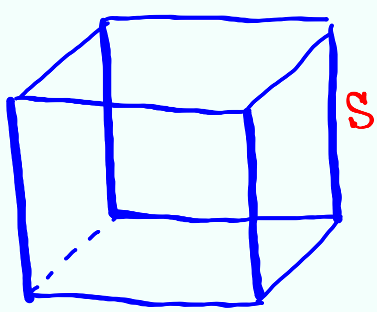
$= \frac{2}{\sqrt{3}}(250)$

$\frac{dx}{dt} = 288$ km/hr.

Speed of car is ≈ 88 km/hr.

Jan 22-11:39 AM

The sides of a cube is increasing at 3cm/s. How fast its volume changing? when side is 2cm.



$V = S^3$

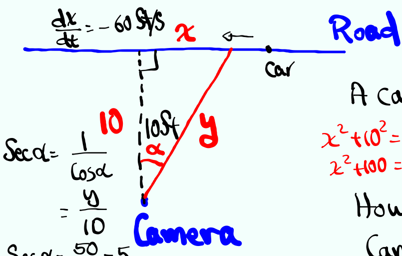
$\frac{dV}{dt} = 3S^2\frac{dS}{dt}$

$= 3(2)^2 \cdot 3$

$= 36 \text{ cm}^3/\text{s}$

Jan 22-11:52 AM

A Camera is 10 ft from a straight road.



A car passes at 60 ft/sec.

$x^2 + 10^2 = y^2$

$x^2 + 100 = 50^2$

$x^2 = 2500 - 100$

$x^2 = 2400$

$x = \sqrt{2400}$

How fast is the Camera's angle changing when the car is 50 ft from it.

$\sec\alpha = \frac{1}{\cos\alpha}$

$= \frac{y}{10}$

$\sec\alpha = \frac{50}{10} = 5$

$\frac{d\alpha}{dt} = ?$ $\quad y = 50$

$\cancel{\sin\alpha = \frac{x}{y}}$ $\quad \cancel{\cos\alpha = \frac{10}{y}}$

$\tan\alpha = \frac{x}{10}$ $\quad \sec^2\alpha \cdot \frac{d\alpha}{dt} = \frac{1}{10} \cdot \frac{dx}{dt}$

$5^2 \cdot \frac{d\alpha}{dt} = \frac{1}{10} \cdot (-60)$

$\frac{d\alpha}{dt} = \frac{-6}{25}$ Rad./Sec.

Jan 22-11:57 AM